Gravity in Randall-Sundrum two D-brane model

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We analyse Randall-Sundrum two D-brane model by linear perturbation and then consider the linearised gravity on the D-brane. The qualitative contribution from the Kaluza-Klein modes of gauge fields to the coupling to the gravity on the brane will be addressed. As a consequence, the gauge fields localised on the brane are shown not to contribute to the gravity on the brane at large distances. Although the coupling between gauge fields and gravity appears in the next order, the ordinary coupling cannot be realised.

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I. INTRODUCTION

Recently the model construction of the inflation using D-brane has been initiated [1] in braneworld context. See Refs. [2, 3, 4] for other related issues. However, the self-gravity of D-brane, which would be essential in considering D-brane cosmology, was not seriously considered there. On the other hand, Randall-Sundrum(RS) [5, 6] type model based on D-brane action has been considered in Refs. [7, 8, 9, 10]. In these papers, the bulk spacetime is described by type IIB supergravity compactified on S^5 [12] and the brane action is the Born-Infeld plus Chern-Simons action. Then the long wave approximation [11] was employed to discuss the low energy effective theory on the D-brane. Although the gauge field was assumed to be localised on the brane, the gravity on the D-brane was shown not to couple to it.

In this paper we will reexamine the gravity on the branes by investigating the linear perturbation. So far, in the previous series of paper on RS D-braneworld [7, 8, 9, 10], the gradient expansion (long wave approximation) has been employed to derive the effective theory on the brane. Furthermore, the form fields B_2 and C_2 are assumed to be closed, that is, $dB_2 = dC_2 = 0$, for simplicity. However, such assumption kills the transverse tensor part of the form fields. In this paper, on the other hand, we will not impose such an assumption. Then we derive the linearised gravity on the D-branes. Our linear perturbation analysis follows Refs. [13, 14, 15, 16]. As seen below, the contribution from the zero mode of gauge fields does not have the usual form and is negligible at large distances. This is consistent with the result from long wave approximation discussed in the previous paper [10] and the appendix of this paper.

The rest of this paper is organised as follows. In Sec. II, we describe the model which we consider here. In Sec. III, we formulate the ADM formalism for the current model. In Sec. IV, the linear perturbation analysis will be done and then derive the linearised gravitational equation on the brane. The contributions from massive modes of the form fields are also considered. Finally we will give summary and discussion in Sec. V. In the appendix, for a comparison with the result obtained in linear perturbation analysis, we rederive the gravitational equation on the brane using the gradient expansion without assumption of $dB_2 = dC_2 = 0$.

II. MODEL

We consider the Randall-Sundrum model in type IIB supergravity compactified on S^5. The brane is described by Born-Infeld and Chern-Simons actions. So we begin with the following action

$$
\begin{aligned}
S &= \frac{1}{2\kappa^2}\int d^5x\sqrt{-\mathcal{G}}\Big[{}^{(5)}R - 2\Lambda - \frac{1}{2}|H|^2 \\
&\quad -\frac{1}{2}(\nabla\chi)^2 - \frac{1}{2}|\tilde{F}|^2 - \frac{1}{2}|\tilde{G}|^2\Big] \\
&\quad + S^{(+)}_{\rm brane} + S^{(+)}_{\rm CS} + S^{(-)}_{\rm brane} + S^{(-)}_{\rm CS}, \qquad (1)
\end{aligned}
$$

where $H_{MNK} = \frac{1}{2}\partial_{[M}B_{NK]}$, $F_{MNK} = \frac{1}{2}\partial_{[M}C_{NK]}$, $G_{K_1K_2K_3K_4K_5} = \frac{1}{4!}\partial_{[K_1}D_{K_2K_3K_4K_5]}$, $\tilde{F} = F + \chi H$ and $\tilde{G} = G + C \wedge H$. $M, N, K = 0, 1, 2, 3, 4$. B_{MN} and C_{MN} are 2-form fields, and $D_{K_1K_2K_3K_4}$ is a 4-form field. χ is a scalar field. $\mathcal{G}_{MN}$ is the metric of five dimensional spacetime.

$S^{(\pm)}_{\rm brane}$ is given by Born-Infeld action

$$S^{(+)}_{\rm brane} = \gamma_{(+)}\int d^4x\sqrt{-\det(h+\mathcal{F}^{(+)})}, \qquad (2)$$

$$S^{(-)}_{\rm brane} = \gamma_{(-)}\int d^4x\sqrt{-\det(q+\mathcal{F}^{(-)})}, \qquad (3)$$

where $h_{\mu\nu}$ and $q_{\mu\nu}$ are the induced metric on the $D_\pm$-brane and

$$\mathcal{F}^{(\pm)}_{\mu\nu} = B^{(\pm)}_{\mu\nu} + (|\gamma_{(\pm)}|)^{-1/2}F^{(\pm)}_{\mu\nu}. \qquad (4)$$

$F_{\mu\nu}$ is the $U(1)$ gauge field on the brane. Here $\mu, \nu = 0, 1, 2, 3$ and $\gamma_{(\pm)}$ are $D_\pm$-brane tension.

$S_{\rm CS}^{(\pm)}$ is Chern-Simons action

$$S_{\rm CS}^{(+)} = \gamma_{(+)}\int d^4x\sqrt{-h}\epsilon^{\mu\nu\rho\sigma}\Bigg[\frac{1}{4}\mathcal{F}^{(+)}_{\mu\nu}C^{(+)}_{\rho\sigma}+\frac{\chi}{8}\mathcal{F}^{(+)}_{\mu\nu}\mathcal{F}^{(+)}_{\rho\sigma} +\frac{1}{24}D^{(+)}_{\mu\nu\rho\sigma}\Bigg], \tag{5}$$

$$S_{\rm CS}^{(-)} = \gamma_{(-)}\int d^4x\sqrt{-q}\epsilon^{\mu\nu\rho\sigma}\Bigg[\frac{1}{4}\mathcal{F}^{(-)}_{\mu\nu}C^{(-)}_{\rho\sigma}+\frac{\chi}{8}\mathcal{F}^{(-)}_{\mu\nu}\mathcal{F}^{(-)}_{\rho\sigma} +\frac{1}{24}D^{(-)}_{\mu\nu\rho\sigma}\Bigg]. \tag{6}$$

Here the brane charges are set equal to the brane tensions. Therefore, our model contains BPS state of D-branes.

III. BASIC EQUATIONS

In this section we write down the basic equations and boundary conditions. Let us perform (1+4)-decomposition

$$ds^2=\mathcal{G}_{MN}dx^M dx^N = e^{2\phi(y,x)}dy^2+g_{\mu\nu}(y,x)dx^\mu dx^\nu, \tag{7}$$

where y is the coordinate orthogonal to the brane. D_+-brane and D_--brane are supposed to locate at $y=y^{(+)}=0$ and $y=y^{(-)}=y_0$.

The spacelike "evolutional" equations to the y-direction are

$$e^{-\phi}\partial_y K = {}^{(4)}R-\kappa^2\left({}^{(5)}T^\mu_\mu-\frac{4}{3}{}^{(5)}T^M_M\right)-K^2 -e^{-\phi}D^2e^{\phi}, \tag{8}$$

$$e^{-\phi}\partial_y\tilde{K}^\mu_\nu = {}^{(4)}\tilde{R}^\mu_\nu-\kappa^2\left({}^{(5)}T^\mu_\nu-\frac{1}{4}\delta^\mu_\nu{}^{(5)}T^\alpha_\alpha\right)-K\tilde{K}^\mu_\nu - e^{-\phi}[D^\mu D_\nu e^{\phi}]_{\rm traceless}, \tag{9}$$

$$\partial_y^2\chi+D^2\chi+e^{\phi}K\partial_y\chi-\frac{1}{2}H_{y\alpha\beta}\tilde{F}^{y\alpha\beta}=0, \tag{10}$$

$$\partial_y X^{y\mu\nu}+e^{\phi}KX^{y\mu\nu}+D_\alpha\phi H^{\alpha\mu\nu}+D_\alpha H^{\alpha\mu\nu} +\frac{1}{2}F_{y\alpha\beta}\tilde{G}^{y\alpha\beta\mu\nu}=0, \tag{11}$$

$$\partial_y \tilde{F}^{y\mu\nu}+e^{\phi}K\tilde{F}^{y\mu\nu}+D_\alpha\phi\tilde{F}^{\alpha\mu\nu}+D_\alpha\tilde{F}^{\alpha\mu\nu} -\frac{1}{2}H_{y\alpha\beta}\tilde{G}^{y\alpha\beta\mu\nu}=0, \tag{12}$$

$$\partial_y\tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4}=e^{\phi}K\tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4}, \tag{13}$$

where $X^{y\mu\nu}:=H^{y\mu\nu}+\chi\tilde{F}^{y\mu\nu}$ and the energy-momentum tensor is

$$\kappa^2\,{}^{(5)}T_{MN}=\frac{1}{2}\left[\nabla_M\chi\nabla_N\chi-\frac{1}{2}g_{MN}(\nabla\chi)^2\right] +\frac{1}{4}\left[H_{MKL}H_N{}^{KL}-g_{MN}|H|^2\right] +\frac{1}{4}\left[\tilde{F}_{MKL}\tilde{F}_N{}^{KL}-g_{MN}|\tilde{F}|^2\right] +\frac{1}{96}\tilde{G}_{MK_1K_2K_3K_4}\tilde{G}_N{}^{K_1K_2K_3K_4}-\Lambda g_{MN}. \tag{14}$$

$K_{\mu\nu}$ is the extrinsic curvature, $K_{\mu\nu}=\frac{1}{2}e^{-\phi}\partial_y g_{\mu\nu}$. $\tilde{K}^\mu_\nu$ and ${}^{(4)}\tilde{R}^\mu_\nu$ are the traceless parts of K^μ_ν and ${}^{(4)}R^\mu_\nu$, respectively. Here D_μ is the covariant derivative with respect to $g_{\mu\nu}$.

The constraints on $y=$ const. hypersurfaces are

$$-\frac{1}{2}\left[{}^{(4)}R-\frac{3}{4}K^2+\tilde{K}^\mu_\nu\tilde{K}^\nu_\mu\right]=\kappa^2\,{}^{(5)}T_{yy}e^{-2\phi}, \tag{15}$$

$$D_\nu K^\nu_\mu-D_\mu K=\kappa^2\,{}^{(5)}T_{\mu y}e^{-\phi}, \tag{16}$$

$$D_\alpha(e^{\phi}X^{y\alpha\mu})+\frac{1}{6}e^{\phi}F_{\alpha_1\alpha_2\alpha_3}\tilde{G}^{y\alpha_1\alpha_2\alpha_3\mu}=0, \tag{17}$$

$$D_\alpha(e^{\phi}\tilde{F}^{y\alpha\mu})-\frac{1}{6}e^{\phi}H_{\alpha_1\alpha_2\alpha_3}\tilde{G}^{y\alpha_1\alpha_2\alpha_3\mu}=0, \tag{18}$$

$$D^\alpha(e^{-\phi}\tilde{G}_{y\alpha\mu_1\mu_2\mu_3})=0. \tag{19}$$

Under Z_2-symmetry, the junction conditions at the brane located $y=y^{(\pm)}$ are

$$\Big[K_{\mu\nu}-g_{\mu\nu}K\Big]_{y=y^{(\pm)}}=\mp\frac{\kappa^2}{2}\gamma_{(\pm)}(g_{\mu\nu}-T^{(\pm)}_{\mu\nu})+O(T^2_{\mu\nu}) \tag{20}$$

$$H_{y\mu\nu}(y^{(\pm)},x)=\mp\kappa^2\gamma_{(\pm)}e^{\phi}\mathcal{F}^{(\pm)}_{\mu\nu}, \tag{21}$$

$$\tilde{F}_{y\mu\nu}(y^{(\pm)},x)=\mp\frac{\kappa^2}{2}\gamma_{(\pm)}e^{\phi}\epsilon_{\mu\nu\alpha\beta}\mathcal{F}^{(\pm)\alpha\beta}, \tag{22}$$

$$\tilde{G}_{y\mu\nu\alpha\beta}(y^{(\pm)},x)=\mp\kappa^2\gamma_{(\pm)}e^{\phi}\epsilon_{\mu\nu\alpha\beta}, \tag{23}$$

$$\partial_y\chi(y^{(\pm)},x)=\mp\frac{\kappa^2}{8}\gamma_{(\pm)}e^{\phi}\epsilon^{\mu\nu\alpha\beta}\mathcal{F}^{(\pm)}_{\mu\nu}\mathcal{F}^{(\pm)}_{\alpha\beta}. \tag{24}$$

In the above

$$T^{(\pm)\mu}{}_\nu=\mathcal{F}^{(\pm)\mu\alpha}\mathcal{F}^{(\pm)}_{\nu\alpha}-\frac{1}{4}\delta^\mu_\nu\mathcal{F}^{(\pm)}_{\alpha\beta}\mathcal{F}^{(\pm)\alpha\beta}. \tag{25}$$

From the junction condition for χ, we can omit the contribution of χ to the gravitational equation on the brane in the approximations which we will employ. Moreover, we omit the quadratic term in Eq. (20).

IV. LINEARISED GRAVITY

A. Background

The background bulk spacetime is five dimensional anti-deSitter spacetime and its metric is given by

$$\overset{(0)}{g}_{\mu\nu}= a^2(y)\eta_{\mu\nu} = e^{-\frac{2y}{\ell}}\eta_{\mu\nu}, \tag{26}$$

where

$$\frac{1}{\ell} = -\frac{1}{6}\kappa^2\gamma_{(+)} = \frac{1}{6}\kappa^2\gamma_{(-)} := -\frac{1}{6}\kappa^2\gamma, \tag{27}$$

and

$$2\Lambda + \frac{5\kappa^4}{6}\gamma^2 = 0. \tag{28}$$

ℓ is the curvature radius of anti-deSitter spacetimes. Eqs. (27) and (28) represent the Randall-Sundrum tuning and then the tension $\gamma_{(+)}$ and $\gamma_{(-)}$ have the same magnitude with opposite signature, $\gamma_{(+)} < 0$ and $\gamma_{(-)} > 0$.

In addition,

$$\tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4} = -a^4\kappa^2\gamma\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}, \tag{29}$$

where $\epsilon_{\alpha_1\alpha_2\alpha_3\alpha_4}$ is the Levi-Civita tensor with respect to the induced metric $h_{\mu\nu}$ on the brane. Other form fields vanish in this order.

B. Linear perturbation

First we consider the linear perturbation for the evolutional equation of $B_{\mu\nu}$ and $C_{\mu\nu}$. In linear order Eqs. (11) and (12) become

$$\partial_y H_{y\mu\nu} + a^{-2}\partial_\alpha H^\alpha{}_{\mu\nu} + \frac{3}{\ell}F_{y\alpha\beta}\epsilon_{\mu\nu}{}^{\alpha\beta} = 0, \tag{30}$$

and

$$\partial_y F_{y\mu\nu} + a^{-2}\partial_\alpha F^\alpha{}_{\mu\nu} - \frac{3}{\ell}H_{y\alpha\beta}\epsilon_{\mu\nu}{}^{\alpha\beta} = 0, \tag{31}$$

where $F^\alpha{}_{\mu\nu} = \eta^{\alpha\beta}F_{\beta\mu\nu}$.

The constraint equations (17) and (18) are rewritten as

$$F_{\mu\nu\alpha} = \frac{\ell}{6}\epsilon_{\mu\nu\alpha}{}^{\beta}\partial^\rho H_{y\rho\beta}, \tag{32}$$

and

$$H_{\mu\nu\alpha} = -\frac{\ell}{6}\epsilon_{\mu\nu\alpha}{}^{\beta}\partial^\rho F_{y\rho\beta}. \tag{33}$$

Here note that we can impose the following gauge conditions

$$B_{y\mu} = C_{y\mu} = 0, \tag{34}$$

using the gauge transformations $B_{MN} \to B'_{MN} = B_{MN} + \partial_M \int_0^y dy' B_{yN}(y',x) - \partial_N \int_0^y dy' B_{yM}(y',x)$ and $C_{MN} \to C'_{MN} = C_{MN} + \partial_M \int_0^y dy' C_{yN}(y',x) - \partial_N \int_0^y dy' C_{yM}(y',x)$. Then

$$H_{y\mu\nu} = \partial_y B_{\mu\nu} \quad \text{and} \quad F_{y\mu\nu} = \partial_y C_{\mu\nu}. \tag{35}$$

$B_{\mu\nu}$ and ${}^*C_{\mu\nu}$ are decomposed to

$$B_{\mu\nu} = B^{TT}_{\mu\nu} + \partial_\mu B^T_\nu - \partial_\nu B^T_\mu, \tag{36}$$

and

$$\begin{aligned} {}^*C_{\mu\nu} &:= \frac{1}{2}\epsilon_{\mu\nu}{}^{\alpha\beta}C_{\alpha\beta} \\ &= {}^*C^{TT}_{\mu\nu} + \partial_\mu {}^*C^T_\nu - \partial_\nu {}^*C^{(T)}_\mu, \end{aligned} \tag{37}$$

where $\partial^\mu B^{TT}_{\mu\nu} = \partial^{\mu *}C^{TT}_{\mu\nu} = \partial^\mu B^T_\mu = \partial^\mu C^T_\mu = 0$.

In momentum space, the field equations are

$$\partial_y^2 B^{(m)TT}_{\mu\nu} - a^{-2}k^2 B^{(m)TT}_{\mu\nu} + \frac{6}{\ell}\partial_y {}^*C^{(m)TT}_{\mu\nu} = 0, \tag{38}$$

$$\partial_y^{2*} C^{(m)TT}_{\mu\nu} + \frac{6}{\ell}\partial_y B^{(m)TT}_{\mu\nu} = 0, \tag{39}$$

$$\partial_y^2 B^{(m)T}_\mu + \frac{6}{\ell}\partial_y {}^*C^{(m)T}_\mu = 0, \tag{40}$$

and

$$\partial_y^{2*} C^{(m)T}_\mu - a^{-2}k^{2*} C^{(m)T}_\mu + \frac{6}{\ell}\partial_y B^{(m)T}_\mu = 0. \tag{41}$$

The constraint equations become

$$B^{(m)TT}_{\mu\nu} + \frac{\ell}{6}\partial_y {}^*C^{(m)TT}_{\mu\nu} = 0, \tag{42}$$

and

$${}^*C^{(m)T}_\mu + \frac{\ell}{6}\partial_y B^{(m)T}_\mu = 0, \tag{43}$$

which are consistent with Eqs. (39) and (40).

Using of Eq. (42), Eq. (38) becomes

$$\partial_y^2 B^{(m)TT}_{\mu\nu} - a^{-2}k^2 B^{(m)TT}_{\mu\nu} - \frac{36}{\ell^2}B^{(m)TT}_{\mu\nu} = 0. \tag{44}$$

In the same way, Eq. (39) with Eq. (43) give us

$$\partial_y^{2*} C^{(m)T}_\mu - a^{-2}k^{2*} C^{(m)T}_\mu - \frac{36}{\ell^2}{}^*C^{(m)T}_\mu = 0. \tag{45}$$

The junction conditions are

$$\partial_y B^{TT}_{\mu\nu}(y^{(\pm)},x) = -\partial_y {}^*C^{TT}_{\mu\nu}(y^{(\pm)},x) = -\kappa^2\gamma B^{(\pm)TT}_{\mu\nu}, \tag{46}$$

and

$$\partial_y B^T_\mu(y^{(\pm)},x) = -\partial_y {}^*C^T_\mu(y^{(\pm)},x) = -\kappa^2\gamma A^{(\pm)T}_\mu. \tag{47}$$

In the above $\mathcal{F}^{(\pm)}_{\mu\nu}$ is decomposed to

$$\begin{aligned}\mathcal{F}^{(\pm)}_{\mu\nu}(x) &= B^{(\pm)TT}_{\mu\nu} + \partial_\mu B^{(\pm)T}_\nu - \partial_\nu B^{(\pm)T}_\mu + F_{\mu\nu} \\ &= B^{(\pm)TT}_{\mu\nu} + \partial_\mu A^{(\pm)T}_\nu - \partial_\nu A^{(\pm)T}_\mu, \end{aligned}\tag{48}$$

where $A^T_\mu := A_\mu + B^T_\mu$ and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$.

Next we focus on the perturbed metric

$$\begin{aligned} ds^2 &= (1+2\phi)dy^2 + (\gamma_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu \\ &= (1+2\phi)dy^2 + (a^2\eta_{\mu\nu} + h^{TT}_{\mu\nu} - \gamma_{\mu\nu}\psi)dx^\mu dx^\nu \end{aligned}\tag{49}$$

where $h^{TT}_{\mu\nu}$ is transverse-traceless part of $h_{\mu\nu}$. Since the bulk background spacetime is anti-deSitter spacetime, the Green function is exactly the same with that of the Randall-Sundrum model. The difference is just presence of the bulk form fields. Therefore we follow the argument of Ref. [13, 14] and then the perturbation $\bar{h}_{\mu\nu}$ in the Gaussian normal coordinate can be computed as

$$\bar{h}_{\mu\nu} = h^{TT}_{\mu\nu} - \gamma_{\mu\nu}\left(\psi - \frac{2}{\ell}\hat{\xi}^5(x)\right), \tag{50}$$

where $\hat{\xi}^5(x)$ is a brane-bending mode (radion field) and

$$\begin{aligned} h^{TT}_{\mu\nu}(y,x) &= -2\kappa^2\int d^4x' G_R(y,x;0,x')|\gamma|\Sigma^{(+)}_{\mu\nu}(x') + 2\kappa^2\int d^4x' G_R(y,x;y_0,x')|\gamma|\Sigma^{(-)}_{\mu\nu}(x') \\ &\quad -2\kappa^2\int dy' d^4x' G_R(y,x;y',x')\delta^{(5)}T_{\mu\nu}(y',x'), \end{aligned}\tag{51}$$

where

$$\Sigma^{(\pm)}_{\mu\nu} = T^{(\pm)}_{\mu\nu} + \frac{1}{\kappa^2}\left(\partial_\mu\partial_\nu\hat{\xi}^{5(\pm)} - \frac{1}{4}\gamma_{\mu\nu}\partial^2\hat{\xi}^{5(\pm)}\right). \tag{52}$$

The first and second terms in the right-hand side of Eq. (51) come from the D_+ and D_- brane, respectively. The third one is the contribution from the bulk fields. In the above $\delta^{(5)}T_{\mu\nu}$ is the projected bulk stress tensor in the linear order. G_R is the five dimensional retarded Green function

$$G_R(y,x;y',x') = G^{(0)}_R(y,x;y',x') + G^{(\mathrm{KK})}_R(y,x;y',x'), \tag{53}$$

where

$$G^{(0)}_R(y,x;y',x') = -\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot(x-x')}\frac{1}{\ell(1-a_0^2)}\frac{a(y)^2a(y')^2}{\mathbf{k}^2-(\omega+i\epsilon)^2}, \tag{54}$$

and

$$G^{(\mathrm{KK})}_R(y,x;y',x') = -\int\frac{d^4k}{(2\pi)^4}e^{ik\cdot(x-x')}\int dm\frac{u_m(y)u_m(y')}{m^2+\mathbf{k}^2-(\omega+i\epsilon)^2}. \tag{55}$$

$G^{(0)}_R$ is the truncated retarded Green function for zero mode. $u_m(y)$ are the mode functions which are expressed by Bessel functions, $u_m(y) \propto J_1(m\ell)N_2(mz) - N_1(m\ell)J_2(mz)$.

In the present model, the equation for $\hat{\xi}^5$ becomes

$$\partial^2\hat{\xi}^5(x) = \frac{\kappa^2}{6}T^{(+)} = 0, \tag{56}$$

that is, the radion is a massless scalar field.

The equation for ψ comes from the Hamiltonian constraint equation:

$$-\frac{3}{2}\frac{1}{a^2}\partial^2\psi = \kappa^2\delta^{(5)}T^y_y, \tag{57}$$

where

$$\begin{aligned}\kappa^2\delta^{(5)}T^y_y &= \frac{1}{8}(H_{y\alpha\beta}H_y{}^{\alpha\beta} + \tilde{F}_{y\alpha\beta}\tilde{F}_y{}^{\alpha\beta}) \\ &\quad -\frac{1}{24}(H_{\mu\alpha\beta}H^{\mu\alpha\beta} + \tilde{F}_{\mu\alpha\beta}\tilde{F}^{\mu\alpha\beta}). \end{aligned}\tag{58}$$

The relation between ϕ and ψ comes from the traceless part of (μ,ν)-component of five dimensional Einstein equation

$$\psi - \phi \sim \kappa^2(1/\partial^2)^2\partial^\mu\partial^\nu[\delta^{(5)}T_{\mu\nu}]_{\mathrm{traceless}}. \tag{59}$$

C. Zero-mode truncation

In order to see the low energy effective (gravitational) theory on the D-brane, we will truncate zero-mode carefully. Let us focus on the zero mode for $B_{\mu\nu}$ and $C_{\mu\nu}$. Introducing new variables

$$\Psi^{(\pm)}_{\mu\nu} := B^{(0)}_{\mu\nu} \pm {}^*C^{(0)}_{\mu\nu}, \tag{60}$$

we obtain two linearly independent equations

$$\partial_y^2 \Psi^{(\pm)}_{\mu\nu} \pm \frac{6}{\ell}\partial_y \Psi^{(\pm)}_{\mu\nu} = 0. \tag{61}$$

The junction conditions are written by

$$\partial_y \Psi^{(+)}_{\mu\nu}(y^{(\pm)}, x) = 0, \tag{62}$$

$$\partial_y \Psi^{(-)}_{\mu\nu}(y^{(\pm)}, x) = -2\kappa^2\gamma\mathcal{F}^{(\pm)}_{\mu\nu}. \tag{63}$$

First it is easy to see that the solutions to the equations for gauge fields are

$$\Psi^{(+)}_{\mu\nu}(y,x) = \alpha_{\mu\nu}(x), \tag{64}$$

and

$$\Psi^{(-)}_{\mu\nu}(y,x) = \frac{2}{\ell}a^{-6}(y)\mathcal{F}^{(+)}_{\mu\nu}(x) + \beta_{\mu\nu}(x), \tag{65}$$

using the junction condition at $y=0$. $\alpha_{\mu\nu}(x)$ and $\beta_{\mu\nu}(x)$ are constant of integrations which is not arisen in $H_{y\mu\nu}$ and $\tilde{F}_{y\mu\nu}$:

$$H^{(0)}_{y\mu\nu}(y,x) = \partial_y B^{(0)}_{\mu\nu}(y,x) = -\kappa^2\gamma a^{-6}(y)\mathcal{F}^{(+)}_{\mu\nu}, \tag{66}$$

$$\tilde{F}^{(0)}_{y\mu\nu}(y,x) = \partial_y C^{(0)}_{\mu\nu}(y,x) = -\frac{\kappa^2\gamma}{2}a^{-6}(y)\epsilon_{\mu\nu}{}^{\alpha\beta}\mathcal{F}^{(+)}_{\alpha\beta}. \tag{67}$$

The remaining junction condition then implies the relation between gauge fields on the two branes

$$a_0^6\mathcal{F}^{(-)}_{\mu\nu} = \mathcal{F}^{(+)}_{\mu\nu}, \tag{68}$$

and

$$T^{(-)}_{0\mu\nu} = a_0^{-14}T^{(+)}_{0\mu\nu}. \tag{69}$$

We also can compute the bulk stress tensor as

$$\begin{aligned}
\kappa^2\delta^{(5)}T_{\mu\nu} &= \frac{1}{2}a^{-2}\left(H_{\mu y\alpha}H_\nu{}^{y\alpha} - \frac{1}{4}\eta_{\mu\nu}H_{y\alpha\beta}H^{y\alpha\beta}\right) + \frac{1}{2}a^{-2}\left(\tilde{F}_{\mu y\alpha}\tilde{F}_\nu{}^{y\alpha} - \frac{1}{4}\eta_{\mu\nu}\tilde{F}_{y\alpha\beta}\tilde{F}^{y\alpha\beta}\right) \\
&\quad + \frac{1}{4}a^{-4}\left(H_{\mu\alpha\beta}H_\nu{}^{\alpha\beta} - \frac{1}{6}\eta_{\mu\nu}H_{\alpha\beta\rho}H^{\alpha\beta\rho}\right) + \frac{1}{4}a^{-4}\left(\tilde{F}_{\mu\alpha\beta}\tilde{F}_\nu{}^{\alpha\beta} - \frac{1}{6}\eta_{\mu\nu}\tilde{F}_{\alpha\beta\rho}\tilde{F}^{\alpha\beta\rho}\right) \\
&= \left(\frac{6}{\ell}\right)^2 a^{-14}T^{(+)}_{\mu\nu} + \frac{1}{4}a^{-4}\left(H_{\mu\alpha\beta}H_\nu{}^{\alpha\beta} - \frac{1}{6}\eta_{\mu\nu}H_{\alpha\beta\rho}H^{\alpha\beta\rho}\right) + \frac{1}{4}a^{-4}\left(\tilde{F}_{\mu\alpha\beta}\tilde{F}_\nu{}^{\alpha\beta} - \frac{1}{6}\eta_{\mu\nu}\tilde{F}_{\alpha\beta\rho}\tilde{F}^{\alpha\beta\rho}\right),
\end{aligned} \tag{70}$$

where $H^{\alpha\beta\rho} = \eta^{\alpha\mu}\eta^{\beta\nu}\eta^{\rho\sigma}H_{\mu\nu\sigma}$. From Eq. (51) with Eqs. (56), (69) and (70) we finally obtain the following linearised equation on branes

$$\begin{aligned}
\partial^2\bar{h}_{\mu\nu}(y,x) &= \partial^2 h^{TT}_{\mu\nu} - \gamma_{\mu\nu}\partial^2\psi \\
&= -2\kappa^2\frac{a^2(y)}{\ell(1-a_0^2)}|\gamma|T^{(+)}_{\mu\nu} + 2\kappa^2\frac{a^2(y)a_0^2}{\ell(1-a_0^2)}|\gamma|T^{(-)}_{\mu\nu} - 2\frac{6}{\ell^2}\frac{a^2(y)(a_0^{-12}-1)}{1-a_0^2}T^{(+)}_{\mu\nu} \\
&\quad -2\frac{a_0^2}{\ell(1-a_0^2)}\int_0^{y_0}dy\frac{1}{4}a^{-2}\left[H_{\mu\alpha\beta}H_\nu{}^{\alpha\beta} + \tilde{F}_{\mu\alpha\beta}\tilde{F}_\nu{}^{\alpha\beta}\right]_{\text{traceless}} + \frac{2}{3}\kappa^2\eta_{\mu\nu}\left(\delta^{(5)}T^y_y\right)^{(0)} \\
&= -2\frac{a_0^2}{\ell(1-a_0^2)}\int_0^{y_0}dy\frac{1}{4}a^{-2}\left[H_{\mu\alpha\beta}H_\nu{}^{\alpha\beta} + \tilde{F}_{\mu\alpha\beta}\tilde{F}_\nu{}^{\alpha\beta}\right]_{\text{traceless}} + \frac{2}{3}\kappa^2\eta_{\mu\nu}\left(\delta^{(5)}T^y_y\right)^{(0)} \\
&= O(H^2_{\mu\nu\alpha}).
\end{aligned} \tag{71}$$

that is, $\partial^2\bar{h}_{\mu\nu}(x) = O(H^2_{\mu\nu\alpha}) = O((\partial_\mu\mathcal{F}_{\nu\alpha})^2)$. The gauge fields localised on the branes does not appear as usual. The appropriate contribution from the boundary stress tensor is exactly cancelled out by that from the bulk stress tensor. In the above $(\cdots)^{(0)}$ represents the zero

mode part. For example, $(\delta^{(5)}T_y^y)^{(0)}$ is

$$(\delta^{(5)}T_y^y)^{(0)} = -\frac{1}{24}(H_{\mu\alpha\beta}H^{\mu\alpha\beta} + \tilde{F}_{\mu\alpha\beta}\tilde{F}^{\mu\alpha\beta}). \quad (72)$$

D. Massive modes of form fields

The mode functions for $B_{\mu\nu}^{(m)TT}$ etc. satisfying the junction condition at $y = y^{(+)} = 0$ is

$$\psi_m = \sqrt{m\ell e^{y/\ell}}\frac{\alpha_m J_6(m\ell e^{y/\ell}) - \beta_m N_6(m\ell e^{y/\ell})}{\sqrt{\alpha_m^2 + \beta_m^2}}, \quad (73)$$

where

$$\alpha_m = m\ell N_5(m\ell) - 6N_6(m\ell), \quad (74)$$

and

$$\beta_m = m\ell J_5(m\ell) - 6J_6(m\ell). \quad (75)$$

m should be quantised by the junction condition at $y = y^{(-)} = y_0$. For $m\ell \gg 1$ and $m\ell e^{y^{(-)}/\ell} \gg 1$, we obtain the mass spectrum of

$$m_n^{(\pm)} \simeq \frac{n\pi}{\ell(1 - e^{y_0/\ell})}. \quad (76)$$

After determination of correct normalisation, for $m\ell \ll 1$, we can evaluate the contribution from massive modes to the right-hand side of Eq. (51) as

$$|\kappa^2\delta^{(5)}T_{\mu\nu}| \simeq \frac{1}{\ell^2}\left(\frac{r_0}{r}\right)^3\left(\frac{\ell}{r}\right)^{12}|T_{\mu\nu}^{(+)}| \ll \frac{1}{\ell^2}|T_{\mu\nu}^{(+)}|, \quad (77)$$

where r_0 is spatial scale of support of form field. Thus even if we consider the contribution from the massive modes, they will be negligible at low energy scale.

V. SUMMARY AND DISCUSSION

In this paper we derived the linearised gravitational equation on the D-brane and then it turned out that the gauge fields do not couple to the gravity on the brane at zero modes in a usual way. Instead, an unusual couplings appear and it is negligible at large distances. We also discussed the contribution from the Kaluza-Klein modes which was shown to be also negligible at large distances.

The model which we considered is minimum extension of Randall-Sundrum type model to the supergravity-like one. Therein Z_2 symmetry and RS tuning are assumed. In this model, RS tuning corresponds to the condition of equality of brane tension and charge. It is likely that D-brane in BPS state does not provide us the realistic model for the braneworld. As analysed in Ref. [9], on the other hand, it was shown that the coupling of the gravity to the gauge fields will appears and the coupling constant is proportional to the cosmological constant. Therefore non-BPS state will be important for braneworld cosmology. We can also consider the cases without Z_2-symmetry. For example, a model considered in Ref. [1] does not have Z_2-symmetry. So the careful study of the self-gravitational effect for such a mode will be important.

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APPENDIX A: LONG-WAVE APPROXIMATION

In this appendix, we approximately solve the bulk field equations by long wave approximation (gradient expansion [11]) and derive the effective theory on the brane. The equation obtained here will include the non-linear effect. Thus we can obtain the same result with one obtained in Sec. IV if we linearise the equation. This appendix can be regarded as the extension of previous work [10] into the general cases where we do not impose $dC_2 = dB_2 = 0$.

In the case with bulk fields we must carefully use the geometrical projection method [17] because the projected Weyl tensor $E_{\mu\nu}$ contains the leading effect from the bulk fields.

The bulk metric is written again as,

$$ds^2 = e^{2\phi(x)}dy^2 + g_{\mu\nu}(y,x)dx^\mu dx^\nu. \quad (A1)$$

The induced metric on the brane will be denoted by $h_{\mu\nu} := g_{\mu\nu}(0,x)$ and then

$$g_{\mu\nu}(y,x) = a^2(y,x)\Big[h_{\mu\nu}(x) + \overset{(1)}{g}_{\mu\nu}(y,x) + \cdots\Big]. \quad (A2)$$

In the above $\overset{(1)}{g}_{\mu\nu}(0,x) = 0$ and $a(0,x) = 1$. In a similar way, the extrinsic curvature is expanded as

$$K^\mu_\nu = \overset{(0)}{K^\mu_\nu} + \overset{(1)}{K^\mu_\nu} + \overset{(2)}{K^\mu_\nu} + \cdots. \quad (A3)$$

The small parameter is $\epsilon = (\ell/L)^2 \ll 1$, where L and ℓ are the curvature scale on the brane and the bulk anti-deSitter curvature scale, respectively.

It is easy to obtain the zeroth order solutions. Without derivation they are given by the Randall-Sundrum set up; Eqs.(27) and (28). Then

$$\overset{(0)}{K^\mu_\nu} = -\frac{1}{\ell}\delta^\mu_\nu, \quad (A4)$$

$$\overset{(0)}{g}_{\mu\nu} = a^2(y,x)h_{\mu\nu}(x) = e^{-\frac{2d(y,x)}{\ell}}h_{\mu\nu}(x), \quad (A5)$$

where

$$d(y,x) = \int_0^y dy e^{\phi(x)}. \qquad \text{(A6)}$$

$\tilde{G}_{y\alpha_1\alpha_2\alpha_3\alpha_4}$ is also given by Eq.(29).

Next we consider the first order equations. The first order equations for $\tilde{F}_{y\mu\nu}$ and $H_{y\mu\nu}$ are

$$\partial_y \overset{(1)}{\tilde{F}}_{y\mu\nu} - \frac{1}{2a^4} \overset{(1)}{H}_{y\alpha\beta} \tilde{G}_{y\rho\sigma\mu\nu} h^{\alpha\rho} h^{\beta\sigma} = 0, \qquad \text{(A7)}$$

and

$$\partial_y \overset{(1)}{H}_{y\mu\nu} + \frac{1}{2a^4} \overset{(1)}{\tilde{F}}_{y\alpha\beta} \tilde{G}_{y\rho\sigma\mu\nu} h^{\alpha\rho} h^{\beta\sigma} = 0. \qquad \text{(A8)}$$

Together with the junction conditions on D_+-brane the solutions are given by

$$\overset{(1)}{H}_{y\mu\nu}(y,x) = -\kappa^2 \gamma a^{-6} e^{\phi} \mathcal{F}^{(+)}_{\mu\nu}, \qquad \text{(A9)}$$

and

$$\overset{(1)}{\tilde{F}}_{y\mu\nu}(y,x) = -\frac{\kappa^2}{2} \gamma a^{-6} e^{\phi} \epsilon_{\mu\nu\rho\sigma} \mathcal{F}^{(+)}_{\alpha\beta} h^{\rho\alpha} h^{\sigma\beta}. \qquad \text{(A10)}$$

The remaining junction conditions on D_--brane imply the relation between $\mathcal{F}^{(+)}_{\mu\nu}$ and $\mathcal{F}^{(-)}_{\mu\nu}$ as

$$\mathcal{F}^{(-)}_{\mu\nu} = a_0^{-6} \mathcal{F}^{(+)}_{\mu\nu}, \qquad \text{(A11)}$$

and then

$$T^{(-)}_{\mu\nu} = a_0^{-14} T^{(+)}_{\mu\nu}, \qquad \text{(A12)}$$

where $a_0 = a(y_0,x) = e^{-d_0(x)/\ell}$ and $d_0(x) := d(y_0,x)$.

Let us first substitute the junction conditions for $H_{y\mu\nu}$ and $\tilde{F}_{y\mu\nu}$ on the D_+ brane into the constraint equations of Eqs. (17) and (18). Then we see

$$\mathcal{D}^{\mu}\left(\mathcal{F}^{(+)}_{\mu\nu} - \frac{1}{2}\epsilon_{\mu\nu\alpha\beta} C^{\alpha\beta} \right) = 0, \qquad \text{(A13)}$$

$$\epsilon^{\mu\nu\alpha\beta} \mathcal{D}_{\nu} (\mathcal{F}^{(+)}_{\alpha\beta} - B_{\alpha\beta}) = 0, \qquad \text{(A14)}$$

where $\mathcal{D}_\mu$ is the covariant derivative with respect to $h_{\mu\nu}$.

Using these results the evolutional equation for the traceless part of the extrinsic curvature is

$$\begin{aligned} e^{-\phi}\partial_y \overset{(1)}{\tilde{K}}{}^{\mu}_{\nu} &= -\overset{(0)}{K}\overset{(1)}{\tilde{K}}{}^{\mu}_{\nu} + \tilde{R}^{\mu}_{\nu}(g) - \kappa^4\gamma^2 a^{-16} T^{(+)\mu}_{\nu} \\ &\quad - e^{-\phi}[D^{\mu}D_{\nu}e^{\phi}]_{\text{traceless}}, \end{aligned} \qquad \text{(A15)}$$

where

$$\tilde{R}^{\mu}_{\nu}(g) = \frac{1}{a^2}\left[R^{\mu}_{\nu}(h) + \frac{2}{\ell}\mathcal{D}^{\mu}\mathcal{D}_{\nu} d + \frac{2}{\ell^2}\mathcal{D}^{\mu} d \mathcal{D}_{\nu} d \right]_{\text{traceless}} \qquad \text{(A16)}$$

and

$$\begin{aligned} D^{\mu}D_{\nu}e^{\phi} &= \frac{1}{a^2}\mathcal{D}^{\mu}\mathcal{D}_{\nu}e^{\phi} + \frac{1}{a^2\ell}\Big(\mathcal{D}^{\mu}e^{\phi}\mathcal{D}_{\nu}d \\ &\quad + \mathcal{D}^{\mu}d\mathcal{D}_{\nu}e^{\phi} - \delta^{\mu}_{\nu}\mathcal{D}^{\alpha}d\mathcal{D}_{\alpha}e^{\phi} \Big). \end{aligned} \qquad \text{(A17)}$$

$R^{\mu}_{\nu}(h) = h^{\mu\alpha}R_{\alpha\nu}(h)$ is the Ricci tensor with respect to $h_{\mu\nu}$ and $T^{\mu}_{\nu} = h^{\mu\alpha}T_{\alpha\nu}$.

The solution is summarised as

$$\begin{aligned} \overset{(1)}{\tilde{K}}{}^{\mu}_{\nu}(y,x) &= -\frac{\ell}{2a^2}{}^{(4)}\tilde{R}^{\mu}_{\nu}(h) + \frac{1}{2}\kappa^2\gamma a^{-16} T^{(+)\mu}_{\nu} \\ &\quad - a^{-2}\left[\mathcal{D}^{\mu}\mathcal{D}_{\nu}d - \frac{1}{\ell}\mathcal{D}^{\mu}d\mathcal{D}_{\nu}d \right]_{\text{traceless}} \\ &\quad + \frac{\chi^{\mu}_{\nu}(x)}{a^4}, \end{aligned} \qquad \text{(A18)}$$

where χ^{μ}_{ν} is the "integration of constant".

The solution to the trace part of the extrinsic curvature is

$$\begin{aligned} \overset{(1)}{K}(y,x) &= -\frac{\ell}{6a^2}{}^{(4)}R(h) \\ &\quad - \frac{1}{a^2}\mathcal{D}^2 d + \frac{1}{a^2\ell}(\mathcal{D}d)^2. \end{aligned} \qquad \text{(A19)}$$

On the D_+-brane Eqs (A18) and (A19) becomes

$${}^{(4)}\tilde{R}^{\mu}_{\nu}(h) = \frac{2}{\ell}\chi^{\mu}_{\nu}(x) \qquad \text{(A20)}$$

and

$$0 = \overset{(1)}{K}(0,x) = -\frac{\ell}{6}{}^{(4)}R(h) \qquad \text{(A21)}$$

They correspond to the Einstein equation on the brane obtained in Ref. [17] and χ^{μ}_{ν} is projected Weyl tensor $E_{\mu\nu}$. For the moment, $\chi^{\mu}_{\nu}(x)$ is unknown term.

On D_--brane, Eq. (A18) becomes

$$\begin{aligned} \frac{\kappa^2}{2}\gamma T^{(-)\mu}_{\nu} &= -\frac{\ell}{2a_0^2}{}^{(4)}\tilde{R}^{\mu}_{\nu}(h) + \frac{\kappa^2}{2}a_0^{-16}\gamma T^{(+)\mu}_{\nu} \\ &\quad - \frac{1}{a_0^2}\left[\mathcal{D}^{\mu}\mathcal{D}_{\nu}d_0 - \frac{1}{\ell}\mathcal{D}^{\mu}d_0\mathcal{D}_{\nu}d_0 \right]_{\text{traceless}} \\ &\quad + \frac{\chi^{\mu}_{\nu}(x)}{a_0^4}. \end{aligned} \qquad \text{(A22)}$$

and

$$0 = \overset{(1)}{K}(y_0,x) = -\frac{1}{a_0^2}\mathcal{D}^2 d_0 + \frac{1}{a_0^2\ell}(\mathcal{D}d_0)^2. \qquad \text{(A23)}$$

All together we obtain the Einstein equation on D_+ brane

$$(a_0^{-2} - 1)G_{\mu\nu}(h) = \frac{2}{\ell}\left[\mathcal{D}_{\mu}\mathcal{D}_{\nu}d_0 - \frac{1}{\ell}\mathcal{D}_{\mu}d_0\mathcal{D}_{\nu}d_0 \right]_{\text{traceless}} \qquad \text{(A24)}$$

The equation for radion becomes

$$\mathcal{D}^2 d_0 - \frac{1}{\ell}(\mathcal{D} d_0)^2 = 0. \tag{A25}$$

Defining

$$\Psi = 1 - e^{-2d_0/\ell} \quad \text{and} \quad \omega(\Psi) = \frac{3}{2}\frac{\Psi}{\Psi - 1}, \tag{A26}$$

we rewrite down the Einstein equation as

$$G_{\mu\nu}(h) = \left[\frac{1}{\Psi}\mathcal{D}_\mu \mathcal{D}_\nu \Psi + \frac{\omega}{\Psi^2}\mathcal{D}_\mu \Psi \mathcal{D}_\nu \Psi\right]_{\rm traceless}. \tag{A27}$$

and

$$\mathcal{D}^2\Psi + \frac{1}{2\omega+3}\frac{d\omega}{d\Psi}(\mathcal{D}\Psi)^2 = 0. \tag{A28}$$

Thus the contribution from the gauge fields to gravity on the brane does not exist at low energy scale.

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